

今まで「等式の性質」
を利用して、方程式を解いてきた。

これからしばらく、

同じ大きさのものは (等値)
入れ替えても (交換)
大きさは、 変わらない。

という考え方に基づいて

[2元1次連立方程式]

を解く。

この考え方を、ふつう

[代入] と呼んでいる。

代入の原理

同じ大きさのものは
入れ替えても
大きさは変わらない。

例えば

$y = x$ ならば

$$\begin{aligned} & 2x + y \\ &= 2x + x \end{aligned}$$

y の代わりに

x を入れるので

「 y に x を代入する」
と言う。

$y = 3x$ ならば

$$\begin{aligned} & 2x + y \\ &= 2x + 3x \end{aligned}$$

y の代わりに

$3x$ を入れるので

「 y に $3x$ を代入する」
と言う。

$y = 5x$ ならば

$$\begin{aligned} & x - y \\ &= x - 5x \end{aligned}$$

y の代わりに

$5x$ を入れるので

「 y に $5x$ を代入する」
と言う。

$$\begin{aligned}
 & x = 3 \text{ のとき} \\
 & x + 2 \\
 = & 3 + 2 \\
 = & 5
 \end{aligned}$$

$$\begin{aligned}
 & x = 5 \text{ のとき} \\
 & x + y \\
 = & 5 + 7 \\
 = &
 \end{aligned}$$

$$\begin{aligned}
 & y = x + 2 \text{ のとき} \\
 & x + y \\
 = & x + x + 2 \\
 = & 2x + 2
 \end{aligned}$$

$$\begin{aligned}
 & x = 5 \text{ のとき} \\
 & x + 2 \\
 = & 5 + 2 \\
 = & 7
 \end{aligned}$$

$$\begin{aligned}
 & x = 10 \text{ のとき} \\
 & x + y \\
 = & 10 + y
 \end{aligned}$$

$$\begin{aligned}
 & y = 2x + 3 \text{ のとき} \\
 & x + y \\
 = & x + 2x + 3 \\
 = & 3x + 3
 \end{aligned}$$

$$\begin{aligned}
 & x = a \text{ のとき} \\
 & x + 2 \\
 = & a + 2
 \end{aligned}$$

$$\begin{aligned}
 & y = x \text{ のとき} \\
 & x + y \\
 = & x + x \\
 = & 2x
 \end{aligned}$$

$$\begin{aligned}
 & y = 2x + 3 \text{ のとき} \\
 & x - y \\
 = & x - (2x + 3) \\
 = & x - 2x - 3 \\
 = & -x - 3
 \end{aligned}$$

$$\begin{aligned}
 & x = 2y \text{ のとき} \\
 & x + 2 \\
 = & 2y + 2
 \end{aligned}$$

$$\begin{aligned}
 & y = 2x \text{ のとき} \\
 & x + y \\
 = & x + 2x \\
 = & 3x
 \end{aligned}$$

$$\begin{aligned}
 & y = 2x - 3 \text{ のとき} \\
 & x - y \\
 = & x - (2x - 3) \\
 = & x - 2x + 3 \\
 = & -x + 3
 \end{aligned}$$

$$\begin{aligned}
 & y = x \text{ のとき} \\
 & 2x + y \\
 = & 2x + x \\
 = & 3x
 \end{aligned}$$

$$\begin{aligned}
 & y = 2x + 1 \text{ のとき} \\
 & x + y \\
 = & x + 2x + 1 \\
 = & 3x + 1
 \end{aligned}$$

$$\begin{aligned}
 & y = 2x \text{ のとき} \\
 & x + y \\
 = & x + 2x \\
 = & 3x
 \end{aligned}$$

$$\begin{aligned}
 & y = x + 1 \text{ のとき} \\
 & x + 2y \\
 = & x + 2(x + 1) \\
 = & x + 2x + 2 \\
 = & 3x + 2
 \end{aligned}$$

$$\begin{aligned}
 & y = x + 1 \text{ のとき} \\
 & x + y \\
 = & x + x + 1 \\
 = & 2x + 1
 \end{aligned}$$

$$\begin{aligned}
 & y = x - 1 \text{ のとき} \\
 & x + 2y \\
 = & x + 2(x - 1) \\
 = & x + 2x - 2 \\
 = & 3x - 2
 \end{aligned}$$

$$\begin{aligned}
 & y = x - 1 \text{ のとき} \\
 & x + y \\
 = & x + x - 1 \\
 = & 2x - 1
 \end{aligned}$$

$$\begin{aligned}
 & y = 2x + 1 \text{ のとき} \\
 & x + 3y \\
 = & x + 3(2x + 1) \\
 = & x + 6x + 3 \\
 = & 7x + 3
 \end{aligned}$$

$$y = 2x + 1 \text{ のとき}$$

$$x - y$$

$$= x - (2x + 1)$$

$$= x - 2x - 1$$

$$= -x - 1$$

$$y = 3x + 1 \text{ のとき}$$

$$x - y$$

$$= x - (3x + 1)$$

$$= x - 3x - 1$$

$$= -2x - 1$$

$$y = 2x - 1 \text{ のとき}$$

$$x - y$$

$$= x - (2x - 1)$$

$$= x - 2x + 1$$

$$= -x + 1$$

$$y = 2x - 3 \text{ のとき}$$

$$x - y$$

$$= x - (2x - 3)$$

$$= x - 2x + 3$$

$$= -x + 3$$

$$y = 2x + 1 \text{ のとき}$$

$$x - 2y$$

$$= x - 2(2x + 1)$$

$$= x - 4x - 2$$

$$= -3x - 2$$

$$y = 3x + 1 \text{ のとき}$$

$$x - 2y$$

$$= x - 2(3x + 1)$$

$$= x - 6x - 2$$

$$= -5x - 2$$

$$y = 2x - 3 \text{ のとき}$$

$$x - 3y$$

$$= x - 3(2x - 1)$$

$$= x - 6x + 9$$

$$= -5x + 9$$

$$y = 3x - 4 \text{ のとき}$$

$$x - 2y$$

$$= x - 2(3x - 4)$$

$$= x - 6x + 8$$

$$= -5x + 8$$

代入の考えで解きなさい。

【例】

$$\begin{cases} x + y = 8 \cdots \textcircled{1} \\ y = 3 \cdots \textcircled{2} \end{cases}$$

①式に②を代入

$$x + 3 = 8$$

$$x = 5$$

$$\begin{cases} x - y = 2 \cdots \textcircled{1} \\ y = 3 \cdots \textcircled{2} \end{cases}$$

①式に②を代入

$$x - 3 = 2$$

$$x = 5$$

$$\begin{cases} x + y = 9 \cdots \textcircled{1} \\ y = 4 \cdots \textcircled{2} \end{cases}$$

①式に②を代入

$$x + 4 = 9$$

$$x = 5$$

$$\begin{cases} x - y = 1 \cdots \textcircled{1} \\ y = 4 \cdots \textcircled{2} \end{cases}$$

①式に②を代入

$$x - 4 = 1$$

$$x = 5$$

$$\begin{cases} x + y = 13 \cdots \textcircled{1} \\ y = 3 \cdots \textcircled{2} \end{cases}$$

①式に②を代入

$$x + 3 = 13$$

$$x = 10$$

$$\begin{cases} x - y = 7 \cdots \textcircled{1} \\ y = 3 \cdots \textcircled{2} \end{cases}$$

①式に②を代入

$$x - 3 = 7$$

$$x = 10$$

「簡単すぎる」と思うだろう。

代入の考えで解きなさい。

$$\begin{cases} 2x + y = 13 \\ y = 3 \end{cases}$$

$$2x + 3 = 13$$

$$2x = 10$$

$$x = 5$$

$$\begin{cases} 2x - y = 7 \\ y = 3 \end{cases}$$

$$2x - 3 = 7$$

$$2x = 10$$

$$x = 5$$

$$\begin{cases} 3x + y = 18 \\ y = 3 \end{cases}$$

$$3x + 3 = 18$$

$$3x = 15$$

$$x = 5$$

$$\begin{cases} 3x - y = 12 \\ y = 3 \end{cases}$$

$$3x - 3 = 12$$

$$2x = 15$$

$$x = 5$$

$$\begin{cases} 5x + y = 23 \\ y = 3 \end{cases}$$

$$5x + 3 = 23$$

$$5x = 20$$

$$x = 4$$

$$\begin{cases} 5x - y = 22 \\ y = 8 \end{cases}$$

$$5x - 8 = 22$$

$$5x = 30$$

$$x = 6$$

簡単過ぎる考え方が、

スゴイ働きにつながるのです。

代入の考えで解きなさい。

【例】

$$\begin{cases} x + 2y = 11 \cdots \textcircled{1} \\ y = x + 1 \cdots \textcircled{2} \end{cases}$$

①式に②を代入

$$x + 2(x + 1) = 11$$

$$x + 2x + 2 = 11$$

$$3x = 9 \cdots \textcircled{3}$$

$$x = 3$$

③を②式に代入

$$y = 3 + 1 = 4$$

$$\begin{cases} x + y = 12 \\ y = x + 2 \end{cases}$$

①式に②を代入

$$x + (x + 2) = 12$$

$$2x + 2 = 12$$

$$2x = 10$$

$$x = 5 \cdots \textcircled{3}$$

③→②

$$y = 5 + 2 = 7$$

$$\begin{cases} x + 4y = 19 \\ y = x + 1 \end{cases}$$

$$x + 4(x + 1) = 19$$

$$x + 4x + 4 = 19$$

$$5x = 15$$

$$x = 3$$

$$y = 3 + 1 = 4$$

$$\begin{cases} x + 3y = 26 \\ y = x + 2 \end{cases}$$

$$x + 3(x + 2) = 26$$

$$x + 3x + 6 = 26$$

$$4x = 20$$

$$x = 5$$

$$y = 7$$

$$\begin{cases} x + 5y = 23 \\ y = x + 1 \end{cases}$$

$$x + 5(x + 1) = 23$$

$$x + 5x + 5 = 23$$

$$6x = 18$$

$$x = 3$$

$$\begin{cases} x + 5y = 40 \\ y = x + 2 \end{cases}$$

$$x + 5(x + 2) = 40$$

$$x + 5x + 10 = 40$$

$$6x = 30$$

$$x = 5$$

$$y = 7$$

代入の考えで解きなさい。

$$\begin{cases} x + y = 7 \\ y = x - 3 \end{cases}$$

$$\begin{cases} x + y = 8 \\ y = x - 2 \end{cases}$$

$$\begin{cases} x + y = 7 \\ y = 2x - 8 \end{cases}$$

$$\begin{cases} x + y = 10 \\ y = x - 2 \end{cases}$$

$$\begin{cases} x + y = 7 \\ y = 3x - 13 \end{cases}$$

$$\begin{cases} x + y = 10 \\ y = 3x - 2 \end{cases}$$

例にならって解きなさい。

【例】

$$\left. \begin{array}{l} 2x + y = 23 \\ +) x - y = 7 \end{array} \right\}$$

$$\begin{array}{l} 3x = 30 \\ x = 10 \\ 20 + y = 23 \\ y = 3 \end{array}$$

$$\left. \begin{array}{l} 3x + y = 34 \\ +) x - y = 6 \end{array} \right\}$$

$$\begin{array}{l} 4x = 40 \\ x = 10 \\ 3 \times 10 + y = 34 \\ y = 4 \end{array}$$

$$\left. \begin{array}{l} 4x + y = 43 \\ +) x - y = 7 \end{array} \right\}$$

$$\begin{array}{l} 5x = 50 \\ x = 10 \\ 4 \times 10 + y = 43 \\ y = 3 \end{array}$$

$$\left. \begin{array}{l} 5x + y = 54 \\ +) x - y = 6 \end{array} \right\}$$

$$\begin{array}{l} 6x = 60 \\ x = 10 \\ 5 \times 10 + y = 54 \\ y = 4 \end{array}$$

$$\left. \begin{array}{l} 3x + 2y = 36 \\ +) x - 2y = 4 \end{array} \right\}$$

$$\begin{array}{l} 4x = 40 \\ x = 10 \\ 3 \times 10 + 2y = 36 \\ 2y = 6 \\ y = 3 \end{array}$$

$$\left. \begin{array}{l} 3x + 2y = 38 \\ +) x - 2y = 2 \end{array} \right\}$$

$$\begin{array}{l} 4x = 40 \\ x = 10 \\ 3 \times 10 + 2y = 38 \\ 2y = 8 \\ y = 4 \end{array}$$

$$\begin{cases} 2a + b = 13 \\ a - b = 2 \quad (+) \end{cases}$$

$$3a = 15$$

$$a = 5$$

$$2 \times 5 + b = 13$$

$$b = 3$$

$$\begin{cases} 2x + y = 14 \\ x - y = 1 \quad (+) \end{cases}$$

$$\begin{cases} 3a + b = 18 \\ a - b = 2 \quad (+) \end{cases}$$

$$\begin{cases} 3x + y = 19 \\ x - y = 1 \quad (+) \end{cases}$$

$$\begin{cases} 4a + b = 23 \\ a - b = 2 \quad (+) \end{cases}$$

$$\begin{cases} 4x + y = 24 \\ x - y = 1 \quad (+) \end{cases}$$

$$\begin{cases} x + y = 13 \\ x - y = 7 \end{cases}$$

$$\begin{cases} x + y = 16 \\ x - y = 4 \end{cases}$$

$$\begin{cases} 2x + y = 23 \\ x - y = 7 \end{cases}$$

$$\begin{cases} 2x + y = 26 \\ x - y = 4 \end{cases}$$

$$\begin{cases} 3x + y = 32 \\ x - y = 8 \end{cases}$$

$$\begin{cases} 3x + y = 36 \\ x - y = 4 \end{cases}$$

$$\begin{cases} x + y = 13 \\ -x + y = 3 \quad (+) \end{cases}$$

$$\begin{aligned} 2y &= 15 \\ y &= 5 \\ x + 8 &= 13 \\ x &= 5 \end{aligned}$$

$$\begin{cases} x + y = 13 \\ -x + y = -1 \quad (+) \end{cases}$$

$$\begin{aligned} 2y &= 12 \\ y &= 6 \\ x + 6 &= 13 \\ x &= 7 \end{aligned}$$

$$\begin{cases} 2x - y = 3 \\ -x + y = 3 \quad (+) \end{cases}$$

$$\begin{aligned} x &= 6 \\ -6 + y &= 3 \\ y &= 9 \end{aligned}$$

$$\begin{cases} 2x - y = 3 \\ -x + y = -1 \quad (+) \end{cases}$$

$$\begin{aligned} x &= 2 \\ -2 + y &= -1 \\ y &= 1 \end{aligned}$$

$$\begin{cases} x + y = 13 \\ -x + y = -3 \quad (+) \end{cases}$$

$$\begin{aligned} 2y &= 10 \\ y &= 5 \\ x + 5 &= 13 \\ x &= 8 \end{aligned}$$

$$\begin{cases} x + y = 13 \\ -x + y = 7 \quad (+) \end{cases}$$

$$\begin{aligned} 2y &= 20 \\ y &= 10 \\ x + 10 &= 3 \\ x &= -7 \end{aligned}$$

$$\begin{cases} 3x + 2y = 21 \dots \textcircled{1} \\ x + y = 8 \dots \textcircled{2} \end{cases}$$

$$\textcircled{2} \times 2 \quad 2x + 2y = 16 \dots \textcircled{2}'$$

$$\textcircled{1} - \textcircled{2}' \quad x = 5 \dots \textcircled{3}$$

$$\textcircled{3} \rightarrow \textcircled{2} \quad 5 + y = 8$$

$$y = 3$$

$$\begin{cases} 3x + 4y = 27 \dots \textcircled{1} \\ x + y = 8 \dots \textcircled{2} \end{cases}$$

$$\textcircled{2} \times 3 \quad 3x + 3y = 24 \dots \textcircled{2}'$$

$$\textcircled{1} - \textcircled{2}' \quad y = 3 \dots \textcircled{3}$$

$$x + 3 = 8$$

$$\textcircled{3} \rightarrow \textcircled{2} \quad x = 5$$

$$\begin{cases} 5x + 2y = 31 \dots \textcircled{1} \\ x + y = 8 \dots \textcircled{2} \end{cases}$$

$$\textcircled{2} \times 2 \quad 2x + 2y = 16 \dots \textcircled{2}'$$

$$\textcircled{1} - \textcircled{2}' \quad 3x = 15$$

$$x = 5 \dots \textcircled{3}$$

$$\textcircled{3} \rightarrow \textcircled{2} \quad 5 + y = 8$$

$$y = 3$$

$$\begin{cases} 3x + 2y = 21 \dots \textcircled{1} \\ x - y = 2 \dots \textcircled{2} \end{cases}$$

$$\textcircled{2} \times 2 \quad 2x - 2y = 4 \dots \textcircled{2}'$$

$$\textcircled{1} + \textcircled{2}' \quad 5x = 25$$

$$x = 5 \dots \textcircled{3}$$

$$\textcircled{3} \rightarrow \textcircled{2} \quad 5 - y = 2$$

$$y = 3$$

$$\begin{cases} 3x + 4y = 27 \dots \textcircled{1} \\ x - y = 2 \dots \textcircled{2} \end{cases}$$

$$\textcircled{2} \times 3 \quad 3x - 3y = 6 \dots \textcircled{2}'$$

$$\textcircled{1} - \textcircled{2}' \quad 7y = 21$$

$$y = 3 \dots \textcircled{3}$$

$$x - 3 = 2$$

$$\textcircled{3} \rightarrow \textcircled{2} \quad x = 5$$

$$\begin{cases} 5x + 2y = 31 \dots \textcircled{1} \\ x - y = 2 \dots \textcircled{2} \end{cases}$$

$$\textcircled{2} \times 5 \quad 5x - 5y = 10 \dots \textcircled{2}'$$

$$\textcircled{1} - \textcircled{2}' \quad 3x = 21$$

$$x = 3 \dots \textcircled{3}$$

$$\textcircled{3} \rightarrow \textcircled{2} \quad x - 3 = 2$$

$$x = 5$$

y の項を一致させて引き算。

$$\begin{cases} 5x + 2y = 31 \\ 2x + y = 13 \end{cases}$$

y の項のプラス マイナスを使って足し算。

$$\begin{cases} 5x + 2y = 31 \\ 2x - y = 7 \end{cases}$$

$$\begin{cases} 5x + 2y = 31 \\ 3x + y = 18 \end{cases}$$

$$\begin{cases} 5x + 2y = 31 \\ 3x - y = 12 \end{cases}$$

$$\begin{cases} 2x + y = 13 \\ 5x + 2y = 31 \end{cases}$$

$$\begin{cases} 3x - y = 12 \\ 5x + 2y = 31 \end{cases}$$

等値交換 (代入) の方法で解きなさい。

$$\begin{cases} y = x - 2 \cdots \textcircled{1} \\ 2x - y = 9 \cdots \textcircled{2} \end{cases}$$

①→②

$$2x - (x - 2) = 9$$

$$2x + x + 2 = 9$$

$$\begin{cases} x = 7 \cdots \textcircled{3} \\ y = 7 - 2 = 5 \cdots \textcircled{4} \end{cases}$$

$$\begin{cases} y = x - 3 \\ 3x - 2y = 14 \end{cases}$$

$$3x - 2(x - 3) = 14$$

$$3x + 2x + 6 = 14$$

$$\begin{cases} x = 8 \\ y = 8 - 3 = 5 \end{cases}$$

$$\begin{cases} y = x - 3 \\ 2x - y = 11 \end{cases}$$

$$2x - (x - 3) = 11$$

$$2x - x + 3 = 11$$

$$\begin{cases} x = 8 \\ y = 8 - 3 = 5 \end{cases}$$

$$\begin{cases} y = x - 3 \\ 6x - 5y = 23 \end{cases}$$

$$6x - 5(x - 3) = 23$$

$$6x - 5x + 15 = 23$$

$$\begin{cases} x = 8 \\ y = 8 - 3 = 5 \end{cases}$$

等値交換 (代入) の方法で解きなさい。

$$\begin{cases} y = x - 20 \\ 2x - y = 90 \end{cases}$$

$$2x - (x - 20) = 90$$

$$2x - x + 20 = 90$$

$$\begin{cases} x = 70 \\ y = 70 - 20 = 50 \end{cases}$$

$$\begin{cases} y = x - 20 \\ 3x - 2y = 100 \end{cases}$$

$$3x - 2(x - 20) = 100$$

$$3x - 2x + 40 = 100$$

$$\begin{cases} x = 60 \\ y = 60 - 20 = 40 \end{cases}$$

$$\begin{cases} y = x - 20 \\ 3x - y = 160 \end{cases}$$

$$3x - (x - 20) = 160$$

$$3x - x + 20 = 160$$

$$2x = 140$$

$$\begin{cases} x = 70 \\ y = 70 - 20 = 50 \end{cases}$$

$$\begin{cases} y = x - 20 \\ 4x - 3y = 130 \end{cases}$$

$$4x - 3(x - 20) = 130$$

$$4x - 3x + 60 = 130$$

$$\begin{cases} x = 70 \\ y = 70 - 20 = 50 \end{cases}$$

等値交換 (代入) の方法で解きなさい。

$$\begin{cases} 2x - y = 8 \\ y = x - 3 \end{cases}$$

$$2x - (x - 3) = 8$$

$$2x - x + 3 = 8$$

$$\begin{cases} x = 5 \\ y = 5 - 3 = 2 \end{cases}$$

$$\begin{cases} 2x - y = 8 \\ y = x - 2 \end{cases}$$

$$2x - (x - 2) = 8$$

$$2x - x + 2 = 8$$

$$\begin{cases} x = 6 \\ y = 6 - 2 = 4 \end{cases}$$

$$\begin{cases} 2x - y = 8 \\ y = x - 5 \end{cases}$$

$$2x - (x - 5) = 8$$

$$2x - x + 5 = 8$$

$$\begin{cases} x = 3 \\ y = 3 - 5 = -2 \end{cases}$$

$$\begin{cases} 3x - y = 13 \\ y = 2x - 8 \end{cases}$$

$$3x - (2x - 8) = 13$$

$$3x - 2x + 8 = 13$$

$$\begin{cases} x = 5 \\ y = 2 \times 5 - 8 = 2 \end{cases}$$

$$\begin{cases} 2x - y = 8 \\ y = 3x - 13 \end{cases}$$

$$2x - (3x - 13) = 8$$

$$2x - 3x + 13 = 8$$

$$-x = -5$$

$$\begin{cases} x = 5 \\ y = 2 \end{cases}$$

$$\begin{cases} 4x - y = 8 \\ y = 2x - 8 \end{cases}$$

$$4x - (2x - 8) = 8$$

$$4x - 2x + 8 = 8$$

$$2x = 0$$

$$\begin{cases} x = 0 \\ y = 2 \times 0 - 8 = -8 \end{cases}$$

等値交換 (代入) の方法で解きなさい。

$$\begin{cases} x - 2y = 4 \\ y = x - 6 \end{cases}$$

$$x - 2(x - 6) = 4$$

$$x - 2x + 12 = 4$$

$$-x = -8$$

$$x = 8$$

$$y = 8 - 6 = 2$$

$$\begin{cases} x - 3y = 2 \\ y = x - 4 \end{cases}$$

$$x - 3(x - 4) = 2$$

$$x - 3x + 12 = 2$$

$$-2x = -10$$

$$x = 5$$

$$y = 5 - 4 = 1$$

$$\begin{cases} x - 2y = 4 \\ y = 2x - 16 \end{cases}$$

$$x - 2(2x - 16) = 5$$

$$x - 4x + 32 = 5$$

$$-3x = -27$$

$$x = 9$$

$$y = 2 \times 9 - 16 = 2$$

$$\begin{cases} x - 3y = 2 \\ y = 2x - 9 \end{cases}$$

$$x - 3(x - 4) = 2$$

$$x - 3x + 12 = 2$$

$$-2x = -10$$

$$x = 5$$

$$y = 5 - 4 = 1$$

$$\begin{cases} x - 2y = 4 \\ y = 3x - 26 \end{cases}$$

$$x - 3(2x - 26) = 7$$

$$x - 6x + 52 = 7$$

$$-5x = -45$$

$$x = 9$$

$$y = 3 \times 9 - 26 = 1$$

$$\begin{cases} x - 3y = 2 \\ y = x - 6 \end{cases}$$

$$x - 3(x - 6) = 2$$

$$x - 3x + 18 = 2$$

$$-2x = -16$$

$$x = 8$$

$$y = 8 - 6 = 2$$